Summer Assignment AP Calculus BC

Calculus is a challenging college level class in mathematics. Calculus is one of the most useful forms of mathematics, and was invented by Sir Isaac Newton in England and Gottfried Wilhelm Leibniz in Germany in order to mathematically model and calculate the changes observable in nature. The subject will stretch your ability to understand the meaning of all of the mathematical functions you have previously studied. Calculus allows for the creation of equations that can model anything from a missile trajectory to the behavior of automobile drivers in traffic. It is used throughout science, engineering and the world of high finance. BC Calculus covers the second semester of the college level calculus series of classes. BC calculus will expand on the basic ideas of calculus from the AB course. There will be new topics in addition to a deeper examination of limits, derivatives and new integral solution techniques. The course will include infinite series – which is a way to build polynomial expressions for any mathematical function. You will learn how to use calculus in both parametric and polar equations. Calculus BC will give you a second chance to earn AB credit (if you didn't pass the exam) as well as the BC credit.

In order to be prepared for this challenging and fast paced course, I have put together a summer assignment package for you to work on. The idea behind this packet is to make sure you practice some of the math that you have learned previously in other math courses. <u>We will not spend any time reviewing pre-calculus for this course when we return in August</u>. The best way to approach the problems in this packet would be to make sure and do a few of them each week of summer. That way you will keep math fresh in your mind and will not have a drop-off before we return in fall. Alternatively, you could do this packet two weeks before school starts to get yourself back up to speed at the end of summer. Either way, this packet will be essential to make sure that you come prepared for calculus class when school resumes. The packet includes an answer key so that you can check your work. Solutions must show work to gain full credit.

In addition to the problems, this assignment also requires that you watch an excellent video series about calculus put together by 3Blue1Brown – an award winning content creator. There are eleven videos and a total run time of almost 3 hours. It may sound like a lot, but each video is only about 20 minutes long, and you will discover some new perspectives on calculus if you have not seen these videos before. I ask that you take brief notes on each video – to prove that you watched them. In the notes – I want you to highlight the most important take-away that you got from watching the video – and the biggest question that the video inspired.

I am excited to get to teach my favorite math course, and hope that you will be excited to learn. This class will require a lot of extra time. The only way to learn this math is through working problem sets and thinking about the math. This will sometimes seem tedious, but there is no substitute for time spent working and building understanding. Your success will depend on how much you put into this class, not your natural math ability. Please take the time to complete this summer assignment and carefully watch the videos. <u>There will be a quiz using some of these exact problems the first day of class – so be warned and be ready!</u>

Enjoy Your Summer

Mr. Finger

3Blue1Brown – Essence of Calculus Video Series

Please watch the following videos, in order. Please take brief notes to show you have watched the videos. Within each set of notes be sure to write down your biggest take-away from the video along with the deepest question the video inspired.

Episode 1 – Essence of Calculus
- https://www.youtube.com/watch?v=WUvTyaaNkzM&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x

Episode 2 – Paradox of the Derivative
https://www.youtube.com/watch?v=9vKqVkMQHKk&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=2

Episode 3 – Derivative Formulas through Geometry
https://www.youtube.com/watch?v=S0_qX4VJhMQ&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=3

Episode 4 – Visualizing the Chain and Product Rule
https://www.youtube.com/watch?v=YG15m2VwSjA&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=4

Episode 5 – What is so Special about e?
https://www.youtube.com/watch?v=m2MIpDrF7Es&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=5

Episode 6 – Implicit Differentiation
https://www.youtube.com/watch?v=qb40J4N1fa4&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=6

Episode 7 – Limits, L'Hopital and Epsilon delta
https://www.youtube.com/watch?v=kfF40MiS7zA&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=7

Episode 8 – Integration and Fundamental Theorem of Calculus
https://www.youtube.com/watch?v=rfG8ce4nNh0&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=8

Episode 9 – What does Area have to do with Slope?
https://www.youtube.com/watch?v=FnJqaIESC2s&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=9

Episode 10 – Higher Order Derivatives
https://www.youtube.com/watch?v=BLkz5LGWihw&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=10

Episode 11 – Taylor Series
https://www.youtube.com/watch?v=3d6DsjIBzJ4&list=PL0-GT3co4r2wlh6UHTUeQsrf3mlS2lk6x&index=11

A: Basic Algebra Skills

A1. **True or false.** If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = \underline{x^{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\underline{\frac{3}{2}}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3} + 2\sqrt{x-3} = 5\sqrt{x-3}$ T F

g. If $(x+3)(x-10) = \underline{2}$, then $x+3 = \underline{2}$ or $x-10 = \underline{2}$. T F

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T: Trigonometry

You should be able to answer these <u>quickly</u>, *without* using calculator and without referring to (or drawing) a unit circle.

T1. Evaluate Trig Functions without a calculator:

1. $\cos\pi$
2. $\sin\dfrac{\pi}{6}$
3. $\sec 210^\circ$
4. $\tan 90^\circ$
5. $\csc(-150)$
6. $\csc\dfrac{3\pi}{2}$
7. $\cos 0$
8. $\sin^{-1}\dfrac{-1}{2}$
9. $\text{Cos}^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$
10. $\tan^{-1}1$
11. $\arcsin 0$
12. $\text{Tan}^{-1}\left(-\sqrt{3}\right)$
13. $\sin\dfrac{2\pi}{3}$
14. $\text{Sin}^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$
15. $\arctan 0$

T2. Find the value of each expression, in exact form.

a. $\sin\frac{2\pi}{3}$

b. $\cos\frac{11\pi}{6}$

c. $\tan\frac{3\pi}{4}$

d. $\sec\frac{5\pi}{3}$

e. $\csc\frac{7\pi}{4}$

f. $\cot\frac{5\pi}{6}$

Note: You will need to know your trig identities, Sum & Difference & Double Angle Formulas:

Memorize the following Trig Identities:

$\sin^2\theta+\cos^2\theta=1$ $\sec^2\theta=1+\tan^2\theta$ $\csc^2\theta=1+\cot^2\theta$

$\sin(-\theta)=-\sin\theta$ $\cos(-\theta)=\cos\theta$ $\tan(-\theta)=-\tan\theta$

$\cos^2\theta=\dfrac{1+\cos 2\theta}{2}$ $\sin^2\theta=\dfrac{1-\cos 2\theta}{2}$

$\sin 2\theta=2\sin\theta\cos\theta$ $\cos 2\theta=\cos^2\theta-\sin^2\theta$

T3 Find the value(s) of x in $[0, 2\pi)$ which solve each equation.

a. $\sin x=\frac{\sqrt{3}}{2}$ **b.** $\cos x=-1$

c. $\tan x=\sqrt{3}$ **d.** $\sec x=-2$

e. $\csc x$ is undefined **f.** $\cot x=1$

T4. Solve the equation. Give *all* real solutions, if any.

a. $\sin 3x=1$ **b.** $2\sqrt{3}\cos(\pi x)=3$

c. $\tan 2x=0$ **d.** $4\sec x+1=9$

e. $\csc(4x+3)=0$ **f.** $3\cot 6x+\sqrt{3}=0$

T5. Solve by factoring. Give *all* real solutions, if any.

a. $4\sin^2 x+4\sin x+1=0$ **b.** $\cos^2 x-\cos x=0$

c. $\sin x\cos x-\sin^2 x=0$ **d.** $x\tan x+3\tan x=x+3$

T6. Graph each function, identifying x- and y-intercepts, if any, and asymptotes, if any.

a. $y=-\sin(2x)$ **b.** $y=4+\cos x$

c. $y=\tan x-1$ **d.** $y=\sec x+1$

e. $y=\csc(\pi x)$ **f.** $y=2\cot x$

S: Solving

S1. Solve by factoring.

a. $x^3 + 5x^2 - x - 5 = 0$

b. $4x^4 + 36 = 40x^2$

c. $(x^3 - 6)^2 + 3(x^3 - 6) - 10 = 0$

d. $x^5 + 8 = x^3 + 8x^2$

S2. Solve by factoring. You should be able to solve each of these *without* multiplying the whole thing out. (In fact, for goodness' sake, please *don't* multiply it all out!)

a. $(x + 2)^2 (x + 6)^3 + (x + 2)(x + 6)^4 = 0$

b. $(2x - 3)^3 (x^2 - 9)^2 + (2x - 3)^5 (x^2 - 9) = 0$

c. $(3x + 11)^5 (x + 5)^2 (2x - 1)^3 + (3x + 11)^4 (x + 5)^4 (2x - 1)^3 = 0$

d. $6x^2 - 5x - 4 = (2x + 1)^2 (3x - 4)^2$

S3. Solve. (*Hint*: Each question *can* be solved by factoring, but there are other methods, too)

a. $a(3a + 2)^{1/2} + 2(3a + 2)^{3/2} = 0$

b. $\sqrt{2x^2 + x - 6} + \sqrt{2x - 3} = 0$

c. $2\sqrt{x + 3} = x + 3$

d. $\dfrac{6}{(2x + 1)^2} + \dfrac{3}{2x + 1} = 1 + \dfrac{2}{2x + 1}$

S4. Solving Inequalities: *Solve and graph the solution*

a. $|x - 3| > 12$

b. $|x - 3| \le 4$

c. $|10x + 8| > 2$

d. $x^2 - 16 < 0$

e. $x^2 + 6x - 16 \le 0$

f. $x^2 - 3x \ge 10$

L: Logarithms and Exponential Functions

L1. Evaluate Logarithms and Exponentials without a calculator

a. $\log_4 64$ **b.** $\log_3 \frac{1}{9}$ **c.** $\log 10$ **d.** $\ln e$

e. $\ln 1$ **f.** $\ln e^3$ **g.** $3^{\log_3 7}$ **h.** $4^{\log_4 \sin x}$

L2. Expand as much as possible.

a. $\ln x^2y^3$ **b.** $\ln \frac{x+3}{4y}$

c. $\ln 3\sqrt{x}$ **d.** $\ln 4xy$

L3. Condense into the logarithm of a single expression.

a. $4\ln x + 5\ln y$ **b.** $\frac{2}{3}\ln a + 5\ln 2$

c. $\ln x - \ln 2$ **d.** $\frac{\ln x}{\ln 2}$

(contrast with part **c**)

L4. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $\ln(x+3) = 2$ **b.** $\ln x + \ln 4 = 1$

c. $\ln x + \ln(x+2) = \ln 3$ **d.** $\ln(x+1) - \ln(2x-3) = \ln 2$

L5. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $e^{4x+5} = 1$ **b.** $2^x = 8^{4x-1}$

c. $100e^{x\ln 4} = 50$ **d.** $2^x = 3^{x-1}$

(need rounded answer only in **d**)

L6. Round final answers to 3 decimal places.

a. At $t = 0$ there were 140 million bacteria cells in a petri dish. After 6 hours, there were 320 million cells. If the population grew exponentially for $t \geq 0$…

…how many cells were in the dish 11 hours after the experiment began?

…after how many hours will there be 1 billion cells?

b. The *half-life* of a substance is the time it takes for half of the substance to decay. The *half-life* of Carbon-14 is 5568 years. If the decay is exponential…

…what percentage of a Carbon-14 specimen decays in 100 years?

…how many years does it take for 90% of a Carbon-14 specimen to decay?

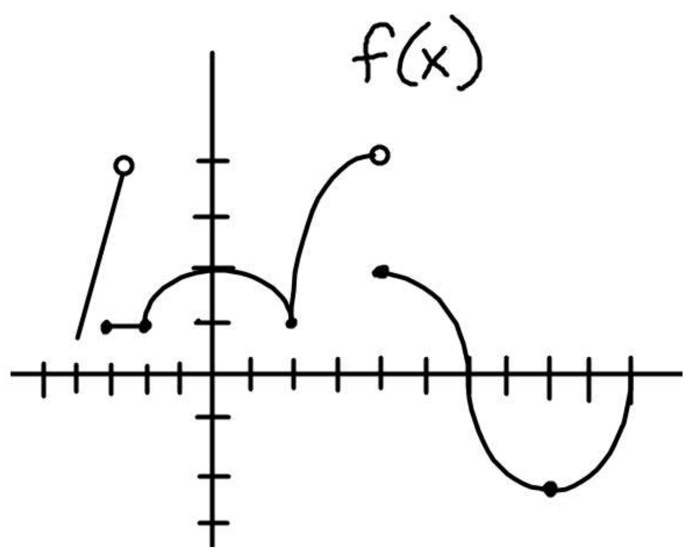
F: FUNCTIONS

Graph each of the following Parent Functions and be familiar with these graphs

1. $f(x) = x$	2. $f(x) = x^2$	3. $f(x) = x^3$
4. $f(x) = \lvert x \rvert$	5. $f(x) = \sqrt{x}$	6. $f(x) = \frac{1}{x}$
7. $f(x) = \frac{1}{x^2}$	8. $f(x) = e^x$	9. $f(x) = \ln x$
10. $f(x) = \sin x$	11. $f(x) = \cos x$	12. $f(x) = \tan x$
13. $f(x) = \tan^{-1} x$	14. $f(x) = x^{\frac{2}{3}}$	15. $f(x) = \frac{1}{1+x^2}$
16. $f(x) = [x]$	17. $f(x) = \sqrt{1 - x^2}$	18. $f(x) = \frac{\lvert x \rvert}{x}$

Analyzing Functions

F1. Increasing/Decreasing



Determine the interval(s) over which f(x) is:

a. Increasing ___________________________

b. Decreasing ___________________________

c. Constant ___________________________

d. Linear ___________________________

e. Concave Up ___________________________

f. What are the zeros of f? _______________

g. For what values of x is f(x) discontinuous? ______________________

F2. Compositions

1. Let $f(x) = 3x^2$ and $g(x) = \frac{x-9}{x+1}$, find the following:

a. $f(g(x))$ b. $g(f(x))$ c. $f^{-1}(x)$

d. Domain, Range, and Zeros of f(x) e. Domain, Range, and Zeros of g(x)

Find f^{-1} and verify that $\left(f \circ f^{-1}\right)(x) = \left(f^{-1} \circ f\right)(x) = x$.

2. $f(x) = 2x + 3$ 3. $f(x) = x^3 - 1$

F3. Piecewise Functions:
Graph and then evaluate the function at the indicated points.

1. $f(x)=\begin{cases} 3x+2, & x>3 \\ -x+4, & x \le 3 \end{cases}$

a. f(2) b. f(3) c. f(5)

2. $f(x)=\begin{cases} x^2\text{-}1, & x<-2 \\ 4, & -2 \le x \le 1 \\ 3x+1, & 1<x<3 \\ x^2-1, & x>3 \end{cases}$

a. f(-3) b. f(-2) c. f(2) d. f(5) e) f(3)

F4. Even/Odd Functions

Show work to determine if the relation is even, odd, or neither.

a. $f(x)=2x^2-7$ **b.** $f(x)=-4x^3-2x$ **c.** $f(x)=4x^2-4x+4$

d. $f(x)=x-\dfrac{1}{x}$ **e.** $f(x)=|x|-x^2+1$ **f.** $f(x)=\sin x+x$

F5. Domains of Functions: Find the Domain of each.

a. $y=\dfrac{3x-2}{4x+1}$ **b.** $y=\dfrac{x^2-4}{2x+4}$ **c.** $y=\dfrac{x^2-5x-6}{x^2-3x-18}$

d. $y=\dfrac{2^{2-x}}{x}$ **e.** $y=\sqrt{x-3}-\sqrt{x+3}$ **f.** $y=\dfrac{\sqrt{2x-9}}{2x+9}$

F6. Asymptotes

Find the equation of both Horizontal and Vertical Asymptotes for the following functions.
Find the coordinates of any holes.

a. $y=\dfrac{x}{x-3}$ **b.** $y=\dfrac{x+4}{x^2-1}$

c. $y=\dfrac{x^2-2x+1}{x^2-3x-4}$ **d.** $y=\dfrac{x^2-9}{x^3-3x^2-18x}$

R: Rational Expressions and Equations

R1.

	Function	Domain	Hole(s): (x, y) if any	Horiz. Asym., if any	Vert. Asym.(s), if any
a.	$f(x)=\dfrac{4x^2+7x-15}{8x^2-14x+5}$				
b.	$f(x)=\dfrac{3(4+x)^2-48}{x}$				
c.	$f(x)=\dfrac{6x+4}{\sqrt{3x^2-10x-8}}$		skip	skip	

R2. Write the equation of a function that has…

a. asymptotes $y = 4$ and $x = 1$, and a hole at (3, 5)

b. holes at (-2, 1) and (2, -1), an asymptote $x = 0$, and no horizontal asymptote

R3. Find the x-coordinates where the function's output is zero and where it is undefined.

a. For what real value(s) of x, if any, is the output of the function $f(x)=\dfrac{x^2+4}{e^{6x}-1}$

…equal to zero? …undefined?

b. For what real value(s) of x, if any, is the output of $g(x)=\dfrac{\cos^2(\pi x)}{\sin x+2}$…

…equal to zero? …undefined?

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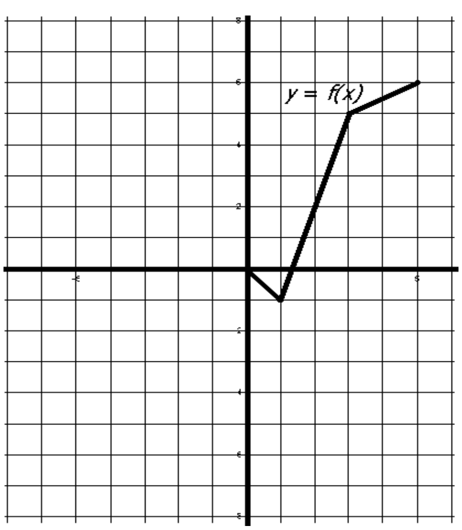
G: Graphing

G1. PART of the graph of f is given. Each gridline represents 1 unit.



a. Complete the graph to make f an EVEN function.

b. What are the domain and range of f_{even}?

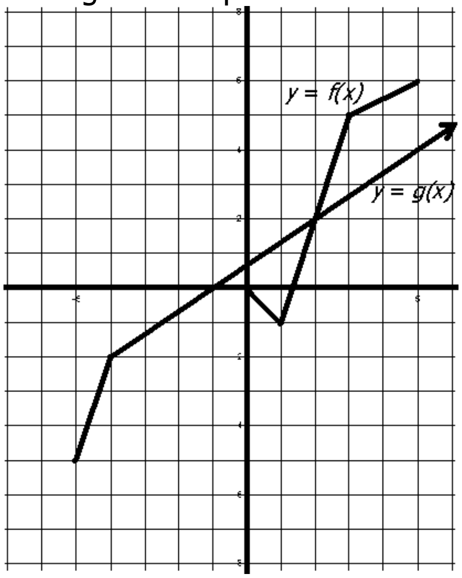
c. What is $f_{even}(-3)$?

d. Complete the graph to make f an ODD function.

e. What are the domain and range of f_{odd}?

f. What is $f_{odd}(-3)$?

G2. The graphs of f and g are given. Answer each question, if possible. If impossible, explain why. Each gridline represents 1 unit.



a. $f^{-1}(5) =$

b. $f(g(5)) =$

c. $(g \circ f)(3) =$

d. Solve for x: $f(g(x)) = 5$

e. Solve for x: $f(x) = g(x)$

For parts **f – i**, respond in interval notation.

f. For what values of x is $f(x)$ increasing?

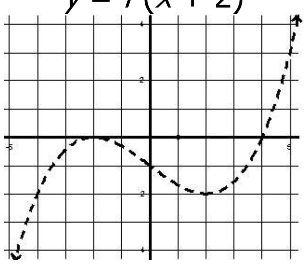
g. For what values of x is $g(x)$ positive?

h. Solve for x: $f(x) < 4$

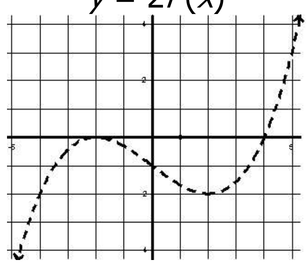
i. Solve for x: $f(x) \geq g(x)$

G3. Given the graph of $y = f(x)$ (dashed graph), sketch each transformed graph.

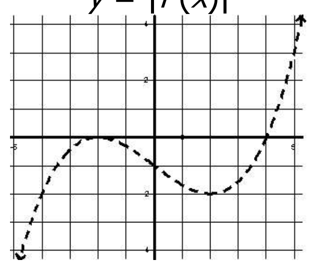
a. $y = f(x + 2)$



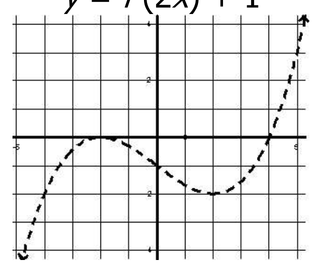
b. $y = 2f(x)$



c. $y = |f(x)|$



d. $y = f(2x) + 1$



PO: Polar Functions

PO1. Plot the point $\left(3, -\frac{3\pi}{4}\right)$ and find three additional represents of this point using $-2\pi < \theta < 2\pi$.

PO2. Convert the given points in polar into rectangular coordinates

(a) $\left(\sqrt{3}, \frac{\pi}{6}\right)$, (b) $\left(2, \frac{2\pi}{3}\right)$, (c) $\left(-3, \frac{-3\pi}{4}\right)$ (d) $\left(-2, \frac{5\pi}{6}\right)$.

PO3. Convert the given points in rectangular into polar coordinates,

(a) (0, 2), (b) $\left(-1, \sqrt{3}\right)$, (c) $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ (d) $\left(\sqrt{3}, -1\right)$.

PO4. Convert the following polar equations into rectangular form:

a. r=2 **b.** $\theta = \frac{\pi}{3}$ **c.** $r = \sec\theta$ **d.** $r = 3\cos\theta + 2\sin\theta$

PO5. Convert the following rectangular equations into polar form

a. y = x **b.** x= 10 **c.** $x^2 + y^2 = 4$ **d.** $x^2 - y^2 = 4x$

PO6. Sketch the graph of the following polar equation:

$r = 3 + 2\cos\theta$

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PA: Parametric Functions

Obtain the rectangular equation by eliminating the parameter. Sketch a graph using the parametric equations:

(a) $x = 2t - 5$, $y = 4t - 7$ (b) $x = 4 - \sqrt{t}$, $y = \sqrt{t}$ (c) $x = t^2$, $y = \sqrt{4 - t^2}$

(d) $x = 4\cos\theta$, $y = 2\sin\theta$ (e) $x = 9\sin^2\theta$, $y = 9\cos^2\theta$ (f) $x = \sec^2\theta - 1$, $y = \tan\theta$

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PF: Partial Fractions

PF1. Find the partial fraction decomposition of

(a) $\dfrac{2x-1}{(x-2)(x-3)}$ (b) $\dfrac{x+7}{x^2 - x - 6}$ (c) $\dfrac{x^2+2}{(x-1)(x+2)(x-3)}$